Vector Revision

Notation (a, b, c) $a\underline{i} + b\underline{j} + c\underline{k}$ $x_1\underline{e}_1 + x_2\underline{e}_2 + x_3\underline{e}_3$

Dot product $\underline{u} = u_1\underline{i} + u_2\underline{j} + u_3\underline{k}$ $\underline{v} = v_1\underline{i} + v_2\underline{j} + v_3\underline{k}$

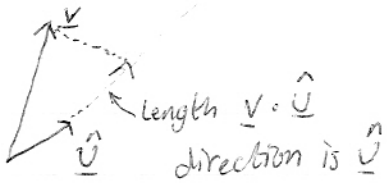
$$\underline{u}\cdot\underline{v} = |\underline{u}||\underline{v}|\cos\theta = u_1v_1 + u_2v_2 + u_3v_3$$

$$\cos\theta = \frac{\underline{u}\cdot\underline{v}}{|\underline{u}||\underline{v}|} \qquad |\underline{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2} \qquad |\underline{u}|^2 = \underline{u}\cdot\underline{u}.$$

$\hat{\underline{u}}$ unit vector $|\hat{\underline{u}}| = 1$ $\hat{\underline{u}} = \frac{\underline{u}}{|\underline{u}|}$

Projection of $\underline{v}$ in the $\hat{\underline{u}}$ direction

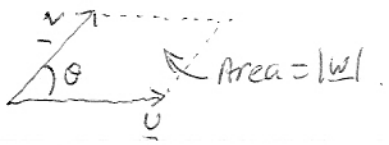
$$(\underline{v}\cdot\hat{\underline{u}})\,\hat{\underline{u}}$$



Cross product $\underline{w} = \underline{u}\times\underline{v}$ is perpendicular to u and v

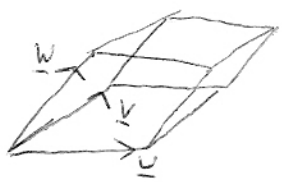
$$|\underline{w}| = |\underline{u}||\underline{v}|\sin\theta$$

$$\underline{v}\times\underline{u} = -\underline{w}$$



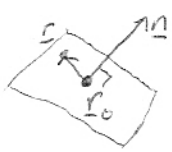
$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = (u_2v_3 - u_3v_2)\underline{i} + (u_3v_1 - u_1v_3)\underline{j} + (u_1v_2 - v_1u_2)\underline{k}$$

Triple product $\underline{u}\cdot(\underline{v}\times\underline{w})$ Signed Volume of (+/- based on whether left or right handed)



A line through $\underline{r}_0$ in the direction $\underline{v}$ is $\underline{r}(t) = \underline{r}_0 + t\underline{v}$ $-\infty < t < \infty$

Equation of a plane A plane through r_0 orthogonal to $\underline{n}$



$$(\underline{r} - \underline{r}_0)\cdot\underline{n} = 0.$$

Vector Valued functions of 1 variable.

A vector $\underline{r}$ which depends on a (real) variable t (say) $\underline{r}(t)$.
Sometimes t will take all real values and sometimes t will be in some interval $a<t<b$ (open interval) $a \leq t \leq b$ (closed interval).

$\underline{r}(t)$ gives the points on some curve.

Tangent to the curve is given by $\underline{V}(t) = \dot{\underline{r}}(t) = \lim_{h \to 0} \left\{ \frac{\underline{r}(t+h) - \underline{r}(t)}{h} \right\}$.

in components $\underline{r}(t) = x(t)\underline{i} + y(t)\underline{j} + z(t)\underline{k}$

then $\dot{\underline{r}}(t) = \dot{x}(t)\underline{i} + \dot{y}(t)\underline{j} + \dot{z}(t)\underline{k}$